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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This is the Sixth Periodic Report on Contract DAJA 45-86-0028; it contains:

1) Statement of the scientific work done during the reporting period

2) Research plans for the immediate future.

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1 Statement of scientific work done during the reporting period

Let $\mathbf{R}^N$ be the N-dimensional real euclidean space, $\underline{x} = (x_1, x_2, \ldots, x_N)^T \in \mathbf{R}^N$ a generic vector, the superscript T means transpose, $<.,.>$ is the euclidean scalar product and $\| \cdot \|$ the euclidean norm. We consider the linear programming problem:

Problem 1 (Linear Programming) Given $\underline{b}, \underline{c} \in \mathbf{R}^N$ and A an $M \times N$ matrix $(M \leq N)$, solve the following minimization problem:

$$
\begin{aligned}
&\text{minimize } \underline{c}, \underline{x} \\
&\text{subject to} \\
&\underline{x} \geq \underline{0} \\
&A\,\underline{x} - \underline{b} \geq \underline{0}
\end{aligned}
$$

Where $\underline{x} \geq \underline{0}$ means that each component of $\underline{x}$ is greater or equal to zero and in a similar way $A\underline{x} - \underline{b} \geq \underline{0}$.

Problem 1 has been reduced to the canonical form necessary to apply Karmarkar's algorithm; for this problem the steepest descent differential equation has been written down. Karmarkar's algorithm corresponds to inte-

grating this differential equation with the Euler-Cauchy scheme with variable stepwise. We are considering the use of A-stable linearly implicit methods to integrate the steepest descent differential equation to improve on Karmarkar's algorithm.

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2 Research Plans for the immediate future

In the future we plan to pursue the following objectives:

(i) study Renegar's method for linear programming in the context of continuation methods

(ii) study the nonlinear complementarity problem.

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Research Projects

Research during the past six months has comprised investigations into a variety of problems, the most relevant of which are contained in the following list.

1. Wedge Entry

This is the problem of the entry of a wedge into water and models the impact of seas on a ship's hull. Dr McLeod, in conjunction with Professor E.G. Fraenkel, whose visit to Oxford was supported by the Grant made an analysis of the integral equation which governs this model, and much qualitative information about the value of possible solutions was obtained. This should lead to a result about the existence of solutions, a question which has been unresolved for some 50 years.

2. One-dimensional Solidification Models

(a) Solidification with kinetic undercooling

The classical Stefan model for supercooled solidification can be stabilised at high solidification rates by the mechanism of kinetic undercooling, but this model has received little mathematical attention except for the weak formulation of Visintin [1]. However, in the one-dimensional, one-phase case, it has now been possible to prove the existence and uniqueness of a classical solution which tends to the well-known solution of the Stefan problem as the kinetic undercooling tends to zero, as long as the solution of the latter problem exists [2].

(b) Analogies with the oxygen-consumption problem

The formal time-derivative of the one-phase supercooled Stefan problem (whose solution can blow up in finite time) is the well-posed oxygen consumption problem and this suggests a mathematical regularisation of the Stefan problem based on integrating the solution of the oxygen consumption. This regularisation and its possible physical implications are listed in [3].

3. Continuous casting

An informal 2-day meeting was arranged in collaboration with Professor D Bland, Cranfield Institute of Technology, in December 1987. As a result of this meeting and the Mathematical Study Group in Heriot-Watt University, March 1988, work is in progress on

(i) the numerical investigation of the continuous dependence of the solution of the classical continuous casting model on the mould heat transfer coefficient;

(ii) flux consumption during continuous casting [4].

4. Explicit Solutions in Two-Dimensional Solidification

Based on earlier work by Ham, some explicit similarity solutions for two-dimensional alloy solidification have been discovered which may be useful for comparison with numerical experiments [5]. Also conformal mapping techniques have been used to solve and unify several related free boundary problems [6].

5. Geometric Model for Dendritic Growth

The famous equation $\epsilon\theta + \theta = \cos\theta$, $\theta(\pm\infty) = \pm \pi/2$ has been studied numerically and analytically, using a high-precision algorithm to deal with cases when ϵ is small (positive or negative). Some preliminary results which confirm non-existence for $\epsilon \neq 0$ are described in [7], [8].

6. Off-axis Foci in Lens Design

As a preliminary to rigorous results concerning the design of axially symmetric lenses with arbitrarily prescribed foci, a formal argument has been prescribed which suggests that when the foci are near to or on the axis of symmetry, the lens must be close to one satisfying the Abbe-Sine or Herschel conditions respectively [9].

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7. Travelling Waves in Neural Networks

This topic has implications both in mathematical biology, the human brain being one of the most obvious examples of a neural network, and in computer science ("connectionism"). Following a one dimensional model constructed by Professor G.B. Ermentrout at the University of Pittsburgh, we have been studying the integro-differential equation

$$+ V = \quad k(x-y)\ S(V(y,t))dy$$

for the potential $V(x,t)$ at a point x on the network and at time t. The given functions k,S reflect the properties of the particular network under discussion.

The most relevant type of solution is a travelling wave (there is evidence, for example, that both migraine and epileptic seizures are travelling waves in the brain), and under suitable conditions on the given functions k, S we have been able to show that the problem possesses a unique travelling wave and that this travelling wave is globally stable. The crucial condition is that S is such that the equation possesses two stable rest-points, say $V \equiv 0$ and $V \equiv 1$.

These results considerably extend and generalise earlier work [1], [2] for the nonlinear diffusion equation

$$u_t = u_{xx} + f(u).$$

8 .Optical Fibre Manufacture

Following earlier work on a simple mathematical model of fibre tapering [3], in which extensional flows of an essentially isoviscous liquid were examined in the absence of surface tension, more general tapering and splicing problems are being considered (in partial collaboration with British Telecom). The single fibre model predicted that nonuniformities in fibre cross section propagated according to a nonlinear wave equation during the tapering process, and a principal goal is to extend this theory to multiple fibres which are being both spliced and tapered.

9. Thermistor Device Modelling

The transient behaviour of circuits containing thermistors is controlled by the heat conduction equation in the thermistor; this involves a nonlinear and nonlocal (in time) Joule heating term. In two dimensions, the steady state solution can often be analysed exactly using complex variables but the evolution problem has only been susceptible to formal asymptotic analysis so far [4,5]. This work has given preliminary indications of both the possibility of rapid temperature surges and the extent of the regions where thermal stresses are likely to be greatest.

10. Semiconductor Fabrication

Work has continued on models of the doping of silicon chips. An initial review of the so-called "birds-beak" phenomenon involving the masking nutride cap has been given in [6] and current research is being devoted to direct oxidation of trenches; the primary goal here is to develop a thermo-elastic model for the stress concentration which can lead to dislocation generation.

11. Phase change Problems

Generalised one-dimensional Stefan problems have been studied with the aim of achieving a better understanding of the stabilising effects of kinetic undercooling [7], and of modelling nucleation in a way which leads to a well posed mathematical model.

9. Other problems and future work

Work has recently begun on study of chemical reactions in hypersonic blunt body flow; the so-called "Newtonian limit" of such flows (in which the specific heat ratio tends to unity) is sufficiently simple that the chemical reaction can be analysed using perturbation theory and an initial paper is being prepared on the topic [8]. It is also anticipated that research will continue on nonlinear wave models for two-phase flow in boilers, based on a recent Ph. D. thesis [9], and on nonlinear diffusion models for ultrafiltration and shear band instabilities.

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